Image-Based Mesh Generation and Volumetric T-spline Modeling for Isogeometric Analysis

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Goal and Motivation

- Developing novel techniques to build high-fidelity computer models at molecular, cellular, tissue and organ scales, and to explore applications in various research fields.

- It is well known that many simulation techniques like finite element method are well-developed and efficient, but mesh generation for complex geometries (e.g., the human body) still takes ~80% of the total analysis time, and generally a great deal of manual interaction is needed.

- Two interdisciplinary research areas with broad applications:
  - Image-based Geometric Modeling and Mesh Generation
  - Volumetric Parameterization for Isogeometric Analysis
  - Biomedical, Material Sciences and Engineering Applications
Image-Based Geometric Modeling and Mesh Generation for Complex Domains
Imaging Data

- Geometric model exists as a level set in volumetric data.
  - Scanned data:
    - CT/MRI/Ultra Sound
    - Cryo-EM
    - EBSD/High energy X-ray
  - Constructed from a function:
    - Signed distance function
    - Electron density map
    - Electron static potential
Image Processing

1. **Contrast enhancement**: design a stretching function to locally change the intensity to reflect better contrast.

2. **Filtering**: bilateral and anisotropic filtering.

3. **Segmentation**: gradient vector diffusion, fast marching, region merging.

4. **Registration**: match two images.

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An Original Slice  
After Filtering  
After Classification  
Segmented Bladder (blue)

Abdominal CT Imaging
Dynamic Lung Modeling and Tumor Tracking Using Deformable Image Registration

- The respiration process can be divided into ten phases;
- The CT image data at each phase were obtained automatically from a 4D CT machine;
- During respiration, the motion of the lung results in the movement of the tumor inside the lung.

- Decrease treatment margin
- Minimize normal tissue exposure

Image-based Mesh Generation

- To automatically generate adaptive and quality boundary (tri/quad) and finite (tet/hex) meshes from volumetric imaging data conforming to boundaries defined as level sets of a scalar field in the volumetric domain. Several finite element mesh generation software packages (e.g., LBIE-Mesher) have been developed.

- Conforming to boundaries
- Adaptive meshes
- Quality meshes
- Correct topology
- All-hex mesh
- Multi-material domain
- Sharp feature preservation
- High order elements
Automatic Mesh Generation for Multi-Material Domain

* Dual Contouring - Minimizer Point Calculation [Ju et al. 2002, Garland and Heckbert 1998]:

\[ QEF[x] = \sum_{i} (n_i \cdot (x - p_i))^2 \]

Sign Change Edge - A sign change edge is an edge whose one vertex lies inside the isocontour (we call it the interior vertex), while the other vertex lies outside.

Material Change Edge - A material change edge is an edge whose two end points lie in two different material regions. A material change edge must be an edge in a boundary cell.

* Tri/Tet meshing:
  - Material change edge
  - Interior edge

* Quad/Hex meshing:
  - Interior grid point
  - Refinement templates
  - Hybrid octree

What is Topology Ambiguity?

- Marching Cubes [Lorensen & Cline 1987] and Dual Contouring [Ju et al. 2002] are able to generate meshes, but they cannot handle topology ambiguities.

What is topology ambiguity? --

One situation where the same sign/material configuration may lead to different topologies in representing the isosurface. For example, diagonally opposite cases have two equally simple possible interpretations.
• After studying the function value at the face saddle points and body saddle points, a total of 31 cases can be found [Natarajan 1994, Lopes & Brodlie 2003].

\[ F(\xi, \eta, \zeta) = F_{000}(1 - \xi)(1 - \eta)(1 - \zeta) + F_{001}(1 - \xi)(1 - \eta)\zeta + F_{010}(1 - \xi)\eta(1 - \zeta) + F_{011}(1 - \xi)\eta\zeta + F_{100}\xi(1 - \eta)(1 - \zeta) + F_{101}\xi(1 - \eta)\zeta + F_{110}\xi\eta(1 - \zeta) + F_{111}\xi\eta\zeta. \]
Multi-Material Domain

- Design an indicator variable to model interior regions

\[ \chi_{i,j} = \begin{cases} 
1 & \text{If } ID_i = \text{material ID of Grid Point } j \\ 
0 & \text{Otherwise} 
\end{cases} \]

- In 3D, topology ambiguities can be removed entirely by tessellating the space using tetrahedra rather than cubes.
- Each face is split into two triangles using the same rule as in 2D ambiguous faces. The cell center and each of these triangles construct a tetrahedron.
Some Ambiguous Cases

- Following the same procedure, we are able to handle any of the ambiguous cases and incorporate correct topologies into triangular and tetrahedral mesh generation.

Quality Improvement and Results

- Vertex classification (7 groups)
- Fairing and regularization for curves
- Face swapping, edge removal, and geometric flow-based smoothing.

**Application: Polycrystalline Materials**

* Develop automatic and robust algorithms and software tools for high-fidelity (correct topology and accurate geometry) geometric modeling and mesh generation for polycrystalline materials.

* Three important features:
  1. Automatic and efficient algorithms of generating meshes directly from voxelized digital data;
  2. Unique capability of generating quality all-hexahedral meshes;
  3. Robust quality improvement and smoothing with the property of volume preserving for each grain.

* Support the understanding, control and optimization of grain boundary dominated materials properties: GBCD/GBED distribution and mechanics characterization

**Collaborators:** CMU-MRSEC, Profs. Tony Rollet and Bob Suter
Crystal plasticity constitutive relation is applied

Compared with brick-shaped meshes: Great reduction in computation time, 4 days VS 8 hours (parallel computing on 156 processors)
High-Order Element Construction: Solid NURBS

- **Image Data**: CT64 data

  - **Image Processing**
    - Contrast Enhancement
    - Filtering
    - Classification
    - Segmentation

  - **Isocontouring & Geometry Editing**
    - Path Extraction (Skeletonization)
    - Skeleton-based Meshing
    - Solid NURBS Construction

  - **Isogeometric Analysis**

  - **Simulation results**

  - **Contrasting & Enhancing**
    - Contouring
    - Surface model
    - Path
    - Control mesh
    - Hex Solid NURBS

Cardiac Hermite Model Construction from Imaging Data

* To develop a NURBS based geometry pipeline for constructing 3D cubic Hermite finite element meshes of the human heart from volumetric imaging data, taking into account the details of the heart structure, including the four main chambers (the left and right atria, ventricles)

Multi-scale Geometric Modeling for Proteins

- Multi-level summation of Gaussian kernel functions – SES surface

\[ G_{i_A}(x) = e^{-\frac{(x-x_{i_A})^2}{2r_{i_A}^2}} \]

\[ G(x) = \sum_{i_C} \left( \sum_{i_R} \left( \sum_{i_A} G_{i_A}(x) \right)^{P_R} \right)^{P_C} \]

\[ \bigcup_{i_R=1}^{n_R} N_R^{(i_R)} = N_A, \quad N_R^{(i_R)} \bigcap_{1 \leq i_R \neq j_R \leq n_R} N_R^{(j_R)} = \emptyset \]

\[ \bigcup_{i_C=1}^{n_C} N_C^{(i_C)} = N_R, \quad N_C^{(i_C)} \bigcap_{1 \leq i_C \neq j_C \leq n_C} N_R^{(j_C)} = \emptyset \]

Input (Protein Data Bank):
- Atom center location;
- Atom radius;
- Hierarchical structure.
Application in Computational Biology: Diffusion Simulation of Biomolecules (Neuro-Muscular Junction)

- mAChE (monomer)
  - Monomeric Mouse acetylcholinesterase (mAChE)
  - Collaborators:
    - Prof. Nathan A. Baker (WUSTL)
    - Prof. J. Andrew McCammon (UCSD)
    - Prof. Michael Holst (UCSD)

- Tetrameric mAChE
  - Tetrameric mAChE

* Smoluchowski Equation:
\[
\frac{\partial c(r,t)}{\partial t} = \nabla \cdot (D(r)c(r)\nabla c(r)) + \beta P(r)\nabla \cdot U(r) = 0
\]
Or in flux operator:
\[
\nabla \cdot \mathbf{j}(r) = 0 \quad \mathbf{j}(r) = D(r)\nabla c(r) + \beta P(r)\nabla U(r)
\]

**Integrals:**
- $\int_{\Omega} \nabla \cdot \mathbf{j}(r) d\Omega = 0$
- $\int_{\Gamma_{u}} n \cdot \mathbf{j}(r) d\Gamma_{u} = 0$
- $\int_{\Gamma_{r}} n \cdot \mathbf{j}(r) d\Gamma_{r} = 0$


Collaborators:
- Prof. Nathan A. Baker (WUSTL)
- Prof. J. Andrew McCammon (UCSD)
- Prof. Michael Holst (UCSD)
Multi-core CPU or GPU-accelerated Multiscale Modeling for Biomolecular Complexes

Number of atoms: \( M \)

Number of grid points: \( N \)

KD-tree or BVH

Neighboring search

\( \log M \)

\( k_A N \)

Time complexity:

\( O(NM) \)

\( O(k_A N \log M) \)

Efficiency improved

Thin filament complex (TFC), atom number: 69,222

2KU2, atom number: 1.23M

To develop an automatic and robust method to generate conformal hex meshes with sharp feature preservation for single-component models and multiple-component assemblies.

- Conformal to the CAD model or assembly
- Good quality (Jacobian and condition #)
- “Fundamental” mesh with no doublet or triangle-shaped quads
- Suitable for finite element simulations

Tri/Tet Mesh Generation with Guaranteed Angle Range

Previous work: Shewchuck’s stuffing: [10.78°, 164.74°] for uniform boundary mesh and [1.66°, 174.72°] for adaptive boundary mesh. Yu’s method: min angle is 5.71° for adaptive mesh.

For given smooth curve or surface, generate adaptive trt/tet mesh with **guaranteed angle range**:
- Triangular mesh: all the angles $\in [19.47°, 141.06°]$;
- Tetrahedral mesh: all the dihedral angle $\in [12.04°, 129.25°]$;
- Interior and exterior meshes with conformal boundary.

Applications ➔ Challenges

Biomedical applications:
• Dynamic modeling and tumor tracking for radiation therapy of lung cancer treatment planning – deformable registration
• Multiscale cardiac modeling for Ca$^{2+}$ signaling in ventricular myocytes, mechanical and electrical activity study
• Vascular blood flow simulation – high-order element (solid NURBS) construction for IGA, estimate wall-thickness and anisotropic material property for FSI, geometry characterization using machine learning
• Dynamic neural foramina cross section measurement and kinematic analysis of lumbar spine undergoing – image segmentation

Material sciences and engineering applications:
• Navy structures - ship design and simulation
• Critical feature determination of polycrystalline microstructure materials
• Nano-scale 3D imaging, meshing, and modeling of fuel cell electrode and other porous materials
• A prototype mesh generation tool development for CFD simulations in architecture domain (GPIC – green buildings)

Various applications give us different challenges on imaging, image processing, geometry processing and simulation.
Volumetric Parameterization (Trivariate T-spline Modeling) for Isogeometric Analysis
Isogeometric Analysis (IGA)

Isogeometric analysis uses the same basis functions to construct geometry and the solution space (FEM + isoparametric). In isogeometric analysis, NURBS/T-spline basis functions are used directly for the analysis.

Why is Isogeometric Analysis needed?

Integrate the engineering design and analysis process and use the spline models directly for analysis. [Hughes et al. 2005]

FEM
- Polynomial basis
- Approximate geometry
- Basis satisfy interpolation property
- Basis have $C^0$ continuity

IGA
- Rational spline basis
- Exact CAD geometry
- Basis do not satisfy interpolation property
- Basis have $C^2$ continuity generally

“The most significant challenge facing isogeometric analysis is developing three-dimensional spline parameterizations from surfaces.” - [Hughes et al. 2009]
NURBS vs T-spline

**T-spline** is a mathematical tool for geometric modeling, which is improved upon and also compatible with NURBS (Non-Uniform Rational B-Spline) [Sederberg et al. 2003].

\[
S(\xi, \eta) = \sum_i P_i w_i B_i(\xi, \eta), \quad B_i(\xi, \eta) = N_i^\xi(\xi) N_i^\eta(\eta)
\]

### NURBS
- Two global knot vectors
- Control points with weight
- Degree

### T-spline
- T-mesh
  - Two local knot vectors for each control point inferred from T-mesh
  - Control points with weight
- Degree
- T-junction
- Local refinement
- Non-rectangular domain

**Other splines:** LR-splines [Dokken & Skytt 2010], PHT-splines [Deng et al. 2008], Modified T-splines [Kang, Chen & Deng 2013], THB-splines [Giannelli et al. 2012].
Converting Unstructured Quad/Hex Mesh to Rational T-Spline

Unstructured Mesh

Topology stage
1. Extraordinary Nodes
2. Sharp Feature Preservation

Valid T-mesh

Geometry stage
1. Surface Fitting
2. Continuity Improvement

Rational T-spline

Rational T-splines are designed to guarantee partition of unity basis functions.

T-spline Surface:

\[ S(\xi, \eta) = \frac{\sum w_i C_i B_i(\xi, \eta)}{\sum w_i B_i(\xi, \eta)}, \quad B_i(\xi, \eta) = N_i^\xi(\xi)N_i^\eta(\eta) \]

Rational T-spline Surface:

\[ S(\xi, \eta) = \frac{\sum w_i C_i R_i(\xi, \eta)}{\sum w_i R_i(\xi, \eta)}, \quad R_i(\xi, \eta) = \frac{\sum N_i^\xi(\xi)N_i^\eta(\eta)}{\sum N_j^\xi(\xi)N_j^\eta(\eta)} \]

Rational solid T-spline:

\[ S(\xi, \eta, \zeta) = \frac{\sum w_i C_i R_i(\xi, \eta, \zeta)}{\sum w_i R_i(\xi, \eta, \zeta)}, \quad R_i(\xi, \eta, \zeta) = \frac{\sum N_i^\xi(\xi)N_i^\eta(\eta)N_i^\zeta(\zeta)}{\sum N_j^\xi(\xi)N_j^\eta(\eta)N_j^\zeta(\zeta)} \]
Irregular Nodes

Table 1. Templates for six quadrilateral element types.

<table>
<thead>
<tr>
<th>Type 0</th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
<th>Type 4</th>
<th>Type 5</th>
</tr>
</thead>
</table>

- **Newly inserted node**
- **Partial extraordinary node**
- **Extraordinary node**

Templates for partial extraordinary nodes in 3D

Templates for extraordinary nodes in 3D
Knot Vector Inference

After we obtain a valid T-mesh, the local knot vectors for each node need to be inferred from the T-mesh.

A regular node or a T-junction

\[ s_A = [0, d_2, d_4, d_4, d_8, d_8, d_8, d_10] \]
\[ t_A = [0, e_1, e_1 + e_3, e_1 + e_2, e_1 + e_2 + e_3] \]

An extraordinary node

\[ s_0^0 = [-e_0, 0, 0, 0, 0] \]
\[ t_0^0 = [-e_1, 0, 0, 0] \]

Nodes adjacent to extraordinary node

\[ s_B^0 = [-e_0, 0, 0, 0, e_2] \]
\[ t_B^0 = [-e_3 - e_1, -e_1, 0, 0, 0] \]
Results

Extraordinary Node #: 4

Input mesh

Bézier elements

Ribosome 30S (quad)
Results
(Solid T-spline)

Statue

Assembly Gear

The constructed solid T-spline and T-mesh

The extracted solid Bézier elements with some elements removed to show the interior mesh.
Solid T-spline for Genus-Zero Geometry

To construct solid rational T-splines for complex genus-zero geometry from the boundary surface triangulation, with $C^2$-continuity everywhere over the boundary surface except for the local region of only eight corner nodes.

$$\sum_{j \in N_i} w_{ij}(f(V_j) - f(V_i)) = 0$$

$$w_{ij} = \cot \alpha_{ij} + \cot \beta_{ij}$$

Quality Improvement

Smoothing: Each node is moved towards the mass center using its neighboring elements;
Optimization: Each node is moved towards an optimal position which maximizes the worst Jacobian.

\[ J = \text{det}(J_M) = \begin{vmatrix} \sum_{i=0}^{7} x_i \frac{\partial N_i}{\partial \xi} & \sum_{i=0}^{7} x_i \frac{\partial N_i}{\partial \eta} & \sum_{i=0}^{7} x_i \frac{\partial N_i}{\partial \zeta} \\ \sum_{i=0}^{7} y_i \frac{\partial N_i}{\partial \xi} & \sum_{i=0}^{7} y_i \frac{\partial N_i}{\partial \eta} & \sum_{i=0}^{7} y_i \frac{\partial N_i}{\partial \zeta} \\ \sum_{i=0}^{7} z_i \frac{\partial N_i}{\partial \xi} & \sum_{i=0}^{7} z_i \frac{\partial N_i}{\partial \eta} & \sum_{i=0}^{7} z_i \frac{\partial N_i}{\partial \zeta} \end{vmatrix} \]

\[ J_s = \frac{J}{\| J_M(\cdot, 0) \| \| J_M(\cdot, 1) \| \| J_M(\cdot, 2) \|} \]

Pillowing & quality improvement result
The input triangle mesh

The mapping result

The subdivision result for the parametric domain

The constructed solid T-spline and T-mesh

The extracted solid Bézier elements

The solid T-spline

Solid Bézier elements with some elements removed

Result - Bunny
Polycube Generation Using Boolean Operations

- Constructive Solid Geometry (CSG) Boolean operations are common in CAD: union, difference and intersection
- Two primitives (cube, torus) and two Boolean operations (union, difference)
- Generate polycubes more flexibly and easily (following the input parameterization directions)

Results – Rod & Assembly

(a) A harmonic field; (b) splitting result; (c) Boolean operations; (d) mapping results; (e) solid T-spline with T-mesh; (f) some elements are removed to show the interior.

Min Jacobian
Rod: 0.34
Assembly: 0.27
Skeleton-based Polycube Construction

- Skeleton generation and splitting – mean curvature flow method [Tagliasacchi et al. 2012];
- Interior cube construction – template to handle branches (bifurcation/trifurcation) [Zhang et al. 2007], enlarge the cross section;
- Boundary cube construction – projecting corner nodes to the surface.

Result - Kitten

- Skeleton
- Singular graph
- Three templates to change the singularity distribution

Input tri mesh
No feature preservation (harmonic field)
With feature preservation
Conformal T-spline Modeling - Boundary Layer Construction

Direction Guidance Using Eigenfunctions – Vector Fields

• Eigenfunctions reflect the object shape \(-\Delta_s f = \lambda f\)
  – The gradient of eigenfunctions follows the structural feature, which can be used to guide the vector field construction;
  – Different modes emphasize different components of the object.

• Feature region of the modes
  – A mode is dominant in a certain region, which we call a feature region;
  – Structural features can be captured by combining multiple feature regions.
Combining Multiple Modes - Characteristic Value

- Represent the feature regions using *feature patches*
  - The surface is subdivided into patches using characteristic values, each patch is assigned to a mode;
  - Guidance triangles are chosen inside each patch.

- Characteristic value - Triangle $T_i$ is assigned to the $k^{th}$ mode if
  $$ k = \arg \max_{j \in \mathbf{K}} C_{i,j} \quad C_{i,j} = \| \nabla F_{i,j} \| $$
Surface Parameterization

- The guidance directions and the cross field [Bommes et al. 2009, Kalberer et al. 2007, Panozzo et al. 2012] are represented as $\gamma_i$ and $\theta_i$ in Triangle $T_i$. Smooth cross field is built based on the guidance directions by minimizing a smoothness energy

$$\Gamma^S = \sum_{e_{ij} \in E} \left( \theta_i + \kappa_{ij} + \frac{\pi}{2} p_{ij} - \theta_j \right)^2 + \lambda \sum_{T_i \in T^g} (\theta_i - \gamma_i)^2$$

where $e_{ij}$ is the edge shared by triangle $T_i$ and $T_j$, $E$ is the set of all the edges in the mesh, $\kappa_{ij}$ is the angle between the reference edges of triangle $T_i$ and $T_j$, and $p_{ij}$ is the integer valued period jump of the cross field across $e_{ij}$.

- To generate a parameterization, the surface is cut into a disk-like region, and the parametric coordinates $(u, v)$ for each vertex can be obtained by minimizing an orientation energy

$$\Gamma^O = \sum_{T_i \in M} A_i \left( \|h \nabla_T u_i - u_i\|^2 + \|h \nabla_T v_i - v_i\|^2 \right)$$

where $h$ is a parameter controlling the spacing of the parametric lines, $\nabla_T u_i$ and $\nabla_T v_i$ are gradients which can be represented by the parametric coordinates $(u, v)$. 
Results

Curvature

Mode 1

\[ N_S = 36 \]

Aneurysm I

Curvature

\[ N_S = 8 \]

Aneurysm II

Eigenfunctions

Modes 1-2

\[ N_S = 50 \]

Thin Filament

Curvature

Modes 1-2

\[ N_S = 32 \]
Truncated Hierarchical Catmull-Clark Subdivision (THCCS)

- Local refinement in Catmull-Clark subdivision

Truncated Hierarchical B-splines

- Truncation mechanism [Giannelli et al. 2012]
  - Discard the children shared with the to-be-refined basis functions

\[ \text{trun}N_i^l = \sum_{\text{passive children}} c_{ij} N_j^{l+1} \]

Basis of THB-splines

Restricted to rectangular parametric domain – can’t handle extraordinary nodes
Construction of THCCS – Refinement

Low-level mesh

Example 1

Example 2

Example 3

Elements

- To-be-refined
- To-be-refined/refined elements
- Example of truncation

Elements not actually refined

- Active children of $B'_1$
- Passive children of $B'_1$

\[
\text{trun}B'_1 = \sum_{\text{Passive children}} c_{ij} B'^{i+1}_j
\]
Construction of THCCS – Truncation

\[ \text{trun}B_i^l = \sum_{\text{Passive children}} c_{ij} B_j^{l+1} \quad \text{subdivision rule} \]

Before truncation

\[ B_1^l \]

Comparison before and after truncation

\[ \text{trun}B_1^l \]

Comparison before and after truncation

\[ \text{trun}B_1^l \]

Comparison before and after truncation

\[ \text{trun}B_1^l \]
Properties of THCCS

• Partition of unity
  – By truncated basis functions
  – Ensures convex hull property
  – Suitable for geometric design

• Global linear independence
  – By selection scheme (active vs passive)

• Geometry preservation
  – Local refinement does not change original geometry
  – Important for both geometric design and analysis
  – Inherit the same continuity as Catmull-Clark subdivision
    • $C^2$-continuous everywhere except $C^1$-continuous at extraordinary nodes
Weighted T-spline - Motivation

**Analysis-suitable T-spline basis** [Scott *et al.* 2012]

- Partition of unity;
- Linear independence [Li *et al.* 2012];
- Convex hull and affine covariance;
- Locally refinability.

**NURBS with local refinement may not be analysis-suitable**

- Partition of unity may be destroyed;
- T-junction extension can be performed to make it analysis-suitable.

---

Left: NURBS surface; Middle: T-spline surface after subdividing one element (partition of unity is not satisfied); Right: standard T-spline [Sederberg *et al.* 2003 & 2004] after T-junction extension.

T-spline generated with four levels of local refinement with T-junction extension to make it analysis-suitable. Left: T-mesh; Right: T-spline.

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**Weighted T-splines** – To enforce T-spline basis functions satisfy partition of unity, we calculate a new weight for each basis function without performing T-junction extension.
Refinability of T-spline basis functions

T-spline basis functions are defined on local knot vectors and can be represented by a linear combination of its children basis functions.

\[ N_i(\xi) = \sum_{p=1}^{n+1} c_{i,p} N_{i,p}^c(\xi) \]

Weighted T-spline basis function

Given the \( j \)th basis function \( N_j^r(\xi,\eta) \) define on the locally refined T-mesh, refinability indicates

\[ N_j^r(\xi,\eta) = \sum_q c_{j,q} N_{j,q}^c(\xi,\eta) \]

The corresponding weighted T-spline basis function is defined as

\[ N_j^w(\xi,\eta) = \sum_q h_{j,q} N_{j,q}^c(\xi,\eta) \]

Based on which criterion to give the weighting coefficient?

PARTITION OF UNITY

Reparameterization of Trimmed NURBS surface

Trimming Curve

Trimming curves are generated by Boolean operations. They are used as a guidance to local refinement of the input NURBS surface, and split the resulting T-mesh elements into preserved elements and removed elements.

Edge Interval Extension

Trimming curve needs to be defined on the boundary of the parametric domain. Connectivity of T-mesh needs to be modified if necessary.

(a) Preserved elements (blue) and removed elements (yellow); (b) the first configuration of preserved elements which does not need a configuration modification; (c) the second configuration of preserved elements that needs a configuration modification; and (d) the connectivity modification result of (c).
### Extraordinary Nodes

#### Requirements
- Gap-free at the extraordinary nodes;
- Gap-free on the spoke edges.

#### Methods
- Apply templates [Wang et al. 2011].
- Capping [Sederberg 2008]
- Constrained optimization framework to directly obtain Bézier control points. [Scott et al. 2013]

#### Design Knot Intervals
- Duplicate the previous knot intervals when meeting the extraordinary nodes.

#### Knot Intervals:
\[ [\xi_3, \xi_3, \xi_3, \xi_4], [\eta_3, \eta_3, \eta_3, \eta_4] \]

Testing Results

Trimming off 2 corners

Trimming off 1 corner

- The trimming curve is preserved exactly;
- The introduced surface error is bounded (< 0.5%), and within three-ring elements around the trimming curve;
- Compared with standard T-spline, the weighted T-spline reduces the number of control points by 19%~31%, and reduces the number of T-mesh elements by 14%~33%.
Rhino 3D to Abaqus: A T-spline Based IGA Software Framework

Current platform includes:
- Data structure conversion
- Volumetric T-spline construction
- Abaqus UELMAT for T-spline based IGA
- Post-processing for results visualization


Hybrid material with plasticity
Summary

Conclusion:
• We have developed schemes to convert any unstructured quad/hex meshes to T-spline surfaces/solids;
• Polycube-based parameteric mapping method is robust for arbitrary-genus objects with feature preservation;
• Truncated hierarchical Catmull-Clark subdivision (THCCS) was developed to handle extraordinary nodes.
• Developed two new generalized T-spline basis functions: weighted T-splines and truncated T-splines

Future Work:
• Analysis-suitable modeling – feature preservation, new basis functions, extraordinary nodes, trimming curves, patch test, optimal convergence rate
• Potential applications in additive manufacturing or 3D printing

Rhino 3D to Abaqus

Trimmed NURBS from CAD Models
Reparameterization
Combination
Construction

T-spline Surfaces
Water-tight T-spline Surfaces
Volumetric T-splines
Conclusion and Discussion

Conclusion:
• Image-based geometric modeling and mesh generation
• Volumetric parameterization for isogeometric analysis
• Biomedical, material sciences, and engineering applications

Discussion and Future Directions:
• Various applications bring us different challenging and interesting research problems
• Image restoration and motion tracking in imaging
• Analysis-suitable models – feature preservation, new basis functions, extraordinary nodes, trimming curves, patch test, optimal convergence rate
• Potential applications in 3D printing
Thanks for your attention!